First Steps in Updating Knowing How

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Overview of the talk

- Background
- The Knowing How logic
- Dynamic modalities: Ontic & epistemic updates
- Conclusions and future work

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- This work: a dynamic epistemic approach of knowing how.
  - Actions updating different kinds of information.

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  1. Ontic updates: modify the ontic information of the models (announcements and arrow updates)
  2. Epistemic updates: modify the perception of the agent about her own abilities (refinements, learning how)

Knowing How: Models

Definition (Uncertainty-based LTS)

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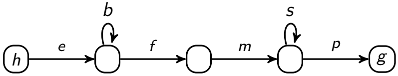

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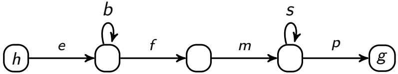

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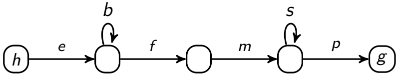

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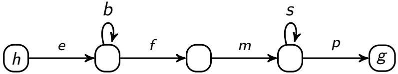

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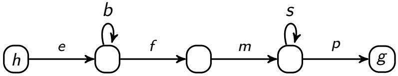

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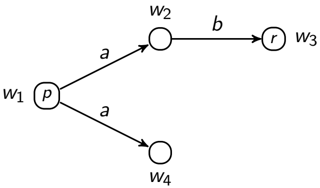

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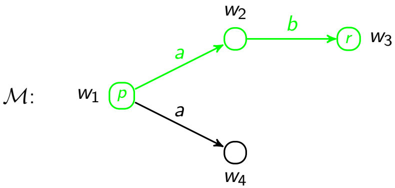

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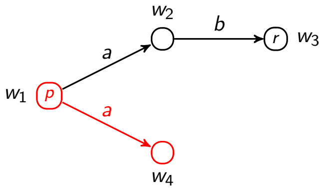

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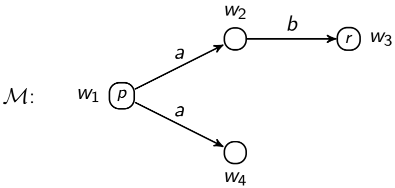

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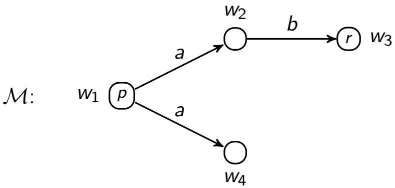

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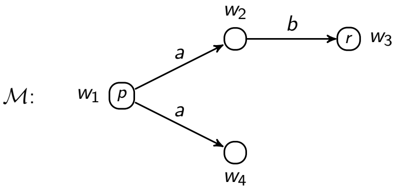

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- $\sigma \in \text{Act}^*$ is SE at a state u iff

  every partial execution of σ from u can be completed.
- $\pi \subseteq \text{Act}^*$ is SE at a state u iff for *all* $\sigma \in \pi$, σ is SE at u.

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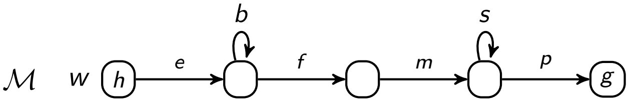

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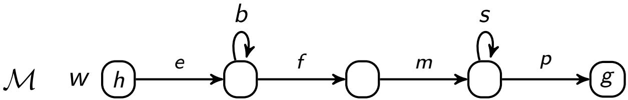

- $\mathbb{S}_i = \{\{ebfmsp\}\}, \qquad \mathbb{S}_j = \{\{ebfmsp, ebmfsp\}\}.$
- $\mathcal{M}, w \models \mathsf{Kh}_i(h, g) \wedge \neg\mathsf{Kh}_j(h, g)$

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This enables us to define ways of updating these two types of information.

Ontic updates: Announcement

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- $\mathsf{V}_{!\chi}(w) = \mathsf{V}(w)$.

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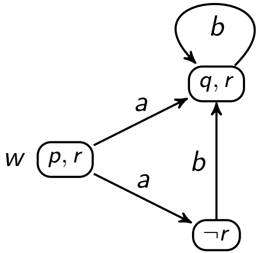

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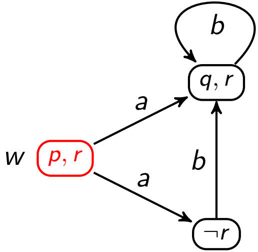

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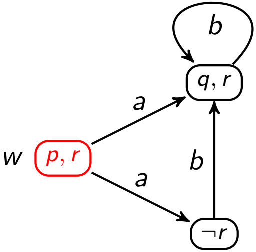

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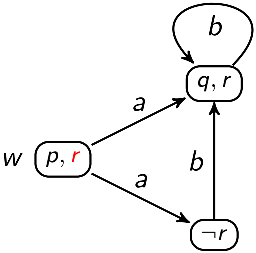

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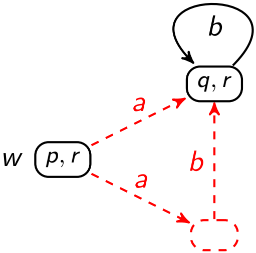

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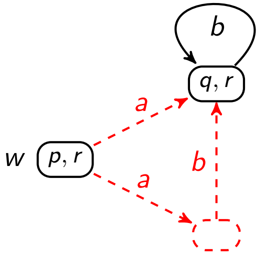

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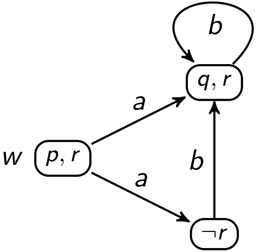

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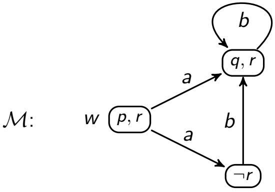

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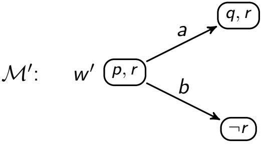

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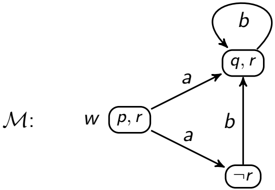

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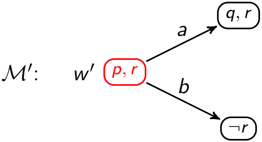

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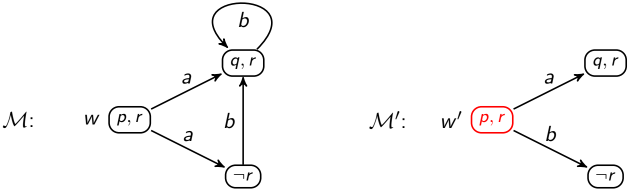

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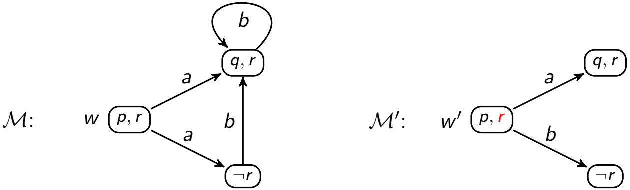

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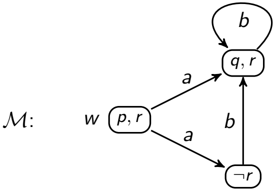

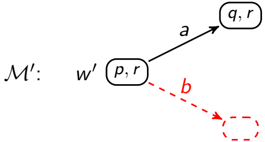

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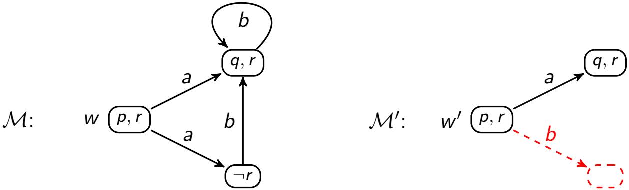

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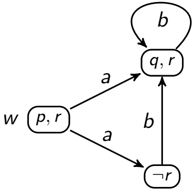

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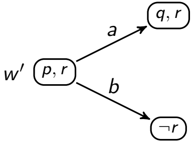

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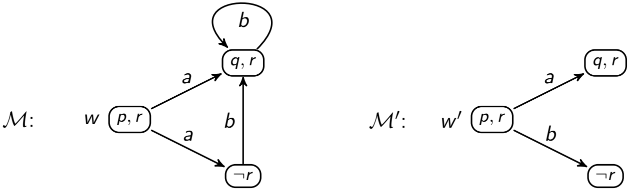

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Epistemic updates: Refinement (L_{Ref})

Definition (L_{Ref} formulas)

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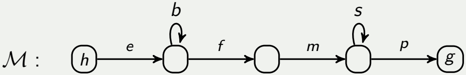

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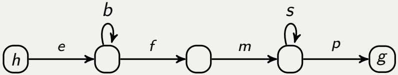
$\mathcal{M}$: 

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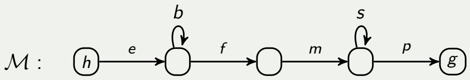

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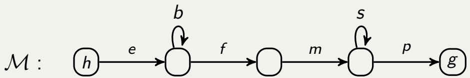

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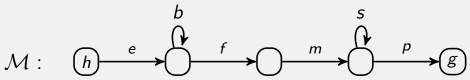

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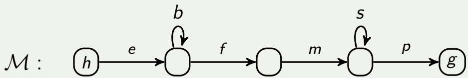

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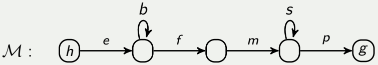

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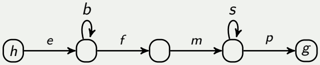

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Conclusions

Dynamic modalities in the context of *knowing how* logics.

- Ontic updates:
  - Announcement-like and arrow-update-like modalities
  - Axiomatizations over a particular class of models
- Epistemic updates:
  - Refining the perception of an agent regarding her own abilities.
  - Preliminary thoughts and some semantic properties.

Future work

- Study other dynamic operators in this context.
- Explore alternative techniques for obtaining proof systems without a general rule of substitution.
- Find fragments that are axiomatizable via reduction axioms by studying the operators' expressivity.